

Checkpoint 6

- (a) Use substitution or elimination to solve the system of equations in Example 6.
- (b) Use technology to solve the system. (Hint: Solve each equation for y , graph both equations on the same screen, and use the intersection finder.)

To find x and y we must solve this system of equations:

$$x + y = 800$$

$$6x + 3y = 3102.$$

The system can readily be solved by hand or by using technology. See Checkpoint 6, which shows that 234 adults and 566 students attended the game.

Chapter 6
Week 2

6.1 Exercises

Determine whether the given ordered list of numbers is a solution of the system of equations.

1. $(-1, 3)$
 $2x + y = 1$
 $-3x + 2y = 9$
2. $(2, -5)$
 $.5x + 8y = -3$
 $x + 5y = -5$

Use substitution to solve each system. (See Example 1.)

3. $3x - y = 1$
 $x + 2y = -9$
4. $x + y = 7$
 $x - 2y = -5$
5. $3x - 2y = 4$
 $2x + y = -1$
6. $5x - 3y = -2$
 $-x - 2y = 3$

Use elimination to solve each system. (See Examples 2–5.)

7. $x - 2y = 5$
 $2x + y = 3$
8. $3x - y = 1$
 $-x + 2y = 4$
9. $2x - 2y = 12$
 $-2x + 3y = 10$
10. $3x + 2y = -4$
 $4x - 2y = -10$
11. $x + 3y = -1$
 $2x - y = 5$
12. $4x - 3y = -1$
 $x + 2y = 19$
13. $2x + 3y = 15$
 $8x + 12y = 40$
14. $2x + 5y = 8$
 $6x + 15y = 18$
15. $2x - 8y = 2$
 $3x - 12y = 3$
16. $3x - 2y = 4$
 $6x - 4y = 8$

In Exercises 17 and 18, multiply both sides of each equation by a common denominator to eliminate the fractions. Then solve the system.

17. $\frac{x}{2} + \frac{y}{3} = 8$
 $\frac{2x}{3} + \frac{3y}{2} = 17$
18. $\frac{x}{5} + 3y = 31$
 $2x - \frac{y}{5} = 8$

19. **Millennials** The number of baby boomers has been decreasing, and the number of millennials has been increasing. Now millennials have surpassed boomers in number. The following equations approximate the population y (in millions) of the

given generation in year x , where $x = 14$ corresponds to the year 2014. (Data from: Pew Research Center.)

$$\text{Boomers: } 10x + 13y = 1125$$

$$\text{Millennials: } -2x + 7y = 495$$

Solve this system of equations, and interpret your answer.

20. **Generation X** The size of generation X has been decreasing, but the size of the baby boomer generation has been decreasing even more rapidly, so the number of generation Xers is projected to overtake the number of baby boomers. The population y (in millions) of the given generation in year x is approximated by

$$\text{Boomers: } 10x + 13y = 1125$$

$$\text{Gen X: } x + 13y = 873,$$

where $x = 14$ corresponds to the year 2014. Solve this system of equations, and interpret your answer. (Data from: Pew Research Center.)

21. **Slow Midwestern Growth** According to U.S. Census Bureau projections through the year 2030, the population y (in millions) of the given state in year x is approximated by

$$\text{Ohio: } -x + 150y = 1710$$

$$\text{North Carolina: } -41x + 300y = 2430,$$

where $x = 0$ corresponds to the year 2000. In what year are the two states projected to have the same population? Round your answer to the nearest year.

22. **Booming Florida** At the start of the millennium, New York was the third most populous state in the country, followed by Florida. Since that time, Florida has experienced faster growth. The population y (in millions) of the given state in year x is approximated by

$$\text{Florida: } 7y - 2x = 112$$

$$\text{New York: } 14y - x = 266,$$

where $x = 0$ corresponds to the year 2000. In what year did Florida overtake New York in population? To the nearest million, what was the population of these states at that time? (Data from: U.S. Census Bureau.)

23. **Google Trends** According to Google Trends, popular interest in light-emitting diode (LED) lightbulbs has been soaring, while interest in compact fluorescent lightbulbs (CFLs) has been dropping. The following equations approximate the Google Trends rating (on a scale from 0 to 100) in year x , where $x = 10$ corresponds to the year 2010. (Data from: www.google.com/trends.)

$$\text{LED: } -25x + 6y = 20$$

$$\text{CFL: } 15x + 2y = 322$$

When did interest in LED lighting surpass CFL lighting? Round your answer to the nearest year.

24. **Heart Disease and Cancer Deaths** The number of deaths y (in thousands) in year x for diseases of the heart and malignant neoplasms (cancer) can be approximated by

$$\text{Heart disease: } 100x + 13y = 9243$$

$$\text{Cancer: } -64x + 26y = 14,379,$$

where $x = 0$ corresponds to the year 2000. According to these models, during what year did deaths from heart disease equal cancer deaths? (Data from: Centers for Disease Control and Prevention.)

25. **Workforce Participation for Women and Men** On the basis of projections for the year 2022, the number of women and men in the workforce (in millions) can be estimated by

$$\text{Women: } -7x + 16y = 1070$$

$$\text{Men: } -5x + 10y = 759,$$

where $x = 14$ corresponds to the year 2014. According to these models, will the number of women in the workforce equal the number of men during the time period from 2014 to 2022 (that is, $14 \leq x \leq 22$)? (Data from: U.S. Department of Labor.)

26. **Weekly Earnings for Women and Men** On the basis of data from the years 2000 through 2015, seasonally adjusted median weekly earnings y in year x can be approximated for women and men by

$$\text{Women: } -791x + 50y = 24,850$$

$$\text{Men: } -84x + 5y = 3210,$$

where $x = 0$ corresponds to the year 2000 and y is in current dollars. If these equations remain valid in the future, will earnings for women catch up with men within the next decade? (Data from: Bureau of Labor Statistics.)

27. **Theater Tickets** A 200-seat theater charges \$8 for adults and \$5 for children. If all seats were filled and the total ticket income was \$1435, how many adults and how many children were in the audience?
28. **Concession Sales** During intermission, the cash bar at a theater sold 72 drinks and took in a total of \$663. If the theater sells beer for \$7 each and wine for \$10 a glass, how many servings of each beverage were sold?
29. **Physical Science** A plane flies 3000 miles from San Francisco to Boston in 5 hours, with a tailwind all the way. The return trip on the same route, now with a headwind, takes 6 hours. Assuming both remain constant, find the speed of the plane and the speed of the wind. [Hint: If x is the plane's speed and y the wind speed (in mph), then the

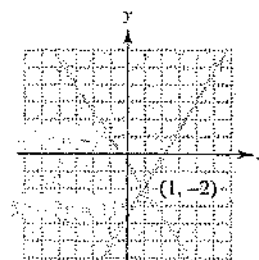
plane travels to Boston at $x + y$ mph because the plane and the wind go in the same direction; on the return trip, the plane travels at $x - y$ mph. (Why?) Then use the equation distance = speed $\cdot$ time.]

30. **Physical Science** A plane flying into a headwind travels 2200 miles in 5 hours. The return flight along the same route with a tailwind takes 4 hours. Find the wind speed and the plane's speed (assuming both are constant). (See the hint for Exercise 29.)

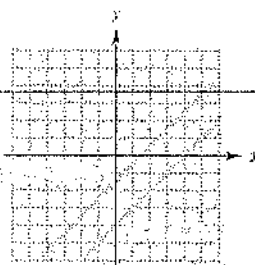
31. (a) Find the equation of the straight line through $(1, 2)$ and $(3, 4)$.
 (b) Find the equation of the line through $(-1, 1)$ with slope 3.
 (c) Find a point that lies on both of the lines in (a) and (b).
32. (a) Find an equation of the straight line through $(0, 9)$ and $(2, 1)$.
 (b) Find an equation of the straight line through $(1, 5)$ with slope 2.
 (c) Find a point that lies on both of the lines in (a) and (b).

Checkpoint Answers

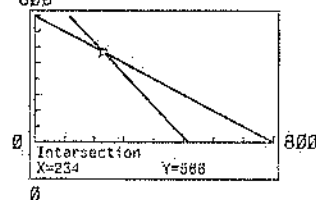
1. $(5, 1)$ 2. $(0, 2)$
 3. $(1, -2)$



4. All ordered pairs that satisfy the equation $3x - 4y = 13$ or $12x - 16y = 52$
 5. No solution



6. (a) $x = 234$ and $y = 566$
 (b) 800



$$A) \begin{bmatrix} 1 & 0 & 0 & 9 & -12 \\ 0 & 1 & 0 & -1 & 4 \\ 0 & 0 & 1 & -2 & 1 \end{bmatrix}$$

Figure 6.12

EXAMPLE 7 Row operations were used to reduce the augmented matrix of a system of three equations in four variables (x , y , z , and w) to the reduced row echelon matrix in Figure 6.12. Solve the system.

Solution First write out the system represented by the matrix:

$$\begin{aligned} x + 9w &= -12 \\ y - w &= 4 \\ z - 2w &= 1 \end{aligned}$$

Let w be the parameter. Solve the first equation for x , the second for y , and the third for z :

$$x = -9w - 12; \quad y = w + 4; \quad z = 2w + 1.$$

The solutions are given by $(-9w - 12, w + 4, 2w + 1, w)$, where w is any real number.

TECHNOLOGY TIP When the system solver on a TI-86 produces an error message, the system might be inconsistent (no solutions) or dependent (infinitely many solutions). You must use RREF or manual methods to solve the system.

The TI-84+ solver usually solves dependent systems directly; Figure 6.13 shows its solutions for Example 7. When the message "no solution found" is displayed, as in Figure 6.14, select RREF at the bottom of the screen to display the reduced row echelon matrix of the system. From that, you can determine the solutions (if there are any) or that no solutions are possible.

Figure 6.13

Figure 6.14

6.2 Exercises

Obtain an equivalent system by performing the stated elementary operation on the system. (See Example 1.)

1. Interchange equations 1 and 2.

$$\begin{aligned} 2x - 4y + 5z &= 1 \\ x - 3z &= 2 \\ 5x - 8y + 7z &= 6 \\ 3x - 4y + 2z &= 3 \end{aligned}$$

2. Interchange equations 1 and 3.

$$\begin{aligned} 2x - 2y + z &= -6 \\ 3x + y + 2z &= 2 \\ x + y - 2z &= 0 \end{aligned}$$

3. Multiply the third equation by $1/2$.

$$\begin{aligned} x + 2y + 4z &= 3 \\ x + 2z &= 0 \\ 2x + 4y + z &= 3 \end{aligned}$$

4. Multiply the first equation by $-\frac{2}{3}$.

$$\begin{aligned} \frac{3}{2}x + 6y - 5z &= 12 \\ 9x + 7z &= 4 \end{aligned}$$

5. Replace the second equation by the sum of itself and -2 times the first equation.

$$\begin{aligned} x + y + 2z + 3w &= 1 \\ 2x + y + 3z + 4w &= 1 \\ 3x + y + 4z + 5w &= 2 \end{aligned}$$

6. Replace the third equation by the sum of itself and -1 times the first equation.

$$\begin{aligned} x + 2y + 4z &= 6 \\ y + z &= 1 \\ x + 3y + 5z &= 10 \end{aligned}$$

7. Replace the third equation by the sum of itself and -2 times the second equation.

$$\begin{aligned}x + 12y - 3z + 4w &= 10 \\2y + 3z + w &= 4 \\4y + 5z + 2w &= 1 \\6y - 2z - 3w &= 0\end{aligned}$$

8. Replace the third equation by the sum of itself and 3 times the second equation.

$$\begin{aligned}2x + 2y - 4z + w &= -5 \\2y + 4z - w &= 2 \\-6y - 4z + 2w &= 6 \\2y + 5z - 3w &= 7\end{aligned}$$

Solve the system by back substitution. (See Example 1.)

$$\begin{aligned}9. \quad x + 3y - 4z + 2w &= 1 \\y + z - w &= 4 \\2z + 2w &= -6 \\3w &= 9\end{aligned}$$

$$\begin{aligned}10. \quad x + 5z + 6w &= 10 \\y + 3z - 2w &= 4 \\z - 4w &= -6 \\2w &= 4\end{aligned}$$

$$\begin{aligned}11. \quad 2x + 2y - 4z + w &= -5 \\3y + 4z - w &= 0 \\2z - 7w &= -6 \\5w &= 15\end{aligned}$$

$$\begin{aligned}12. \quad 3x - 2y - 4z + 2w &= 6 \\2y + 5z - 3w &= 7 \\3z + 4w &= 0 \\3w &= 15\end{aligned}$$

Write the augmented matrix of each of the given systems. Do not solve the systems. (See Example 2.)

$$\begin{aligned}13. \quad 2x + y + z &= 3 & 14. \quad 3x + 4y - 2z - 3w &= 0 \\3x - 4y + 2z &= 5 & x - 3y + 7z + 4w &= 9 \\x + y + z &= 2 & 2x + 5z - 6w &= 0\end{aligned}$$

Write the system of equations associated with each of the given augmented matrices. Do not solve the systems.

$$\begin{aligned}15. \quad \left[\begin{array}{ccc|c} 2 & 3 & 8 & 20 \\ 1 & 4 & 6 & 12 \\ 0 & 3 & 5 & 10 \end{array} \right] & 16. \quad \left[\begin{array}{ccc|c} 3 & 2 & 6 & 18 \\ 2 & -2 & 5 & 7 \\ 1 & 0 & 5 & 20 \end{array} \right]\end{aligned}$$

Use the indicated row operation to transform each matrix. (See Examples 2 and 3.)

17. Interchange R_1 and R_3 .

$$\left[\begin{array}{ccc|c} 5 & -1 & 4 & 8 \\ 2 & 2 & 9 & 3 \\ 1 & 7 & -3 & 5 \end{array} \right]$$

18. Interchange R_2 and R_3 .

$$\left[\begin{array}{ccc|c} 1 & 8 & 3 & -2 \\ 0 & 7 & -9 & 3 \\ 0 & 1 & 8 & -4 \end{array} \right]$$

19. Replace R_3 by $\frac{1}{4}R_3$.

$$\left[\begin{array}{ccc|c} 2 & 5 & 1 & -1 \\ -4 & 0 & 4 & 6 \\ 6 & 0 & 8 & -4 \end{array} \right]$$

20. Replace R_1 with $-\frac{1}{6}R_1$.

$$\left[\begin{array}{ccc|c} -6 & 3 & -8 & 12 \\ 7 & 4 & -5 & 25 \end{array} \right]$$

21. Replace R_2 by $2R_1 + R_2$.

$$\left[\begin{array}{cccc|c} -4 & -3 & 1 & -1 & 2 \\ 8 & 2 & 5 & 0 & 6 \\ 0 & -2 & 9 & 4 & 5 \end{array} \right]$$

22. Replace R_3 by $-3R_1 + R_3$.

$$\left[\begin{array}{cccc|c} 1 & 5 & 2 & 0 & -1 \\ 8 & 5 & 4 & 6 & 6 \\ 3 & 0 & 7 & 1 & -4 \end{array} \right]$$

In Exercises 23–28, the reduced row echelon form of the augmented matrix of a system of equations is given. Find the solutions of the system. (See Example 3.)

$$23. \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/2 \\ 0 & 1 & 0 & 0 & 17 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$24. \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$25. \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$26. \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$27. \quad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 12 \\ 0 & 1 & 0 & 2 & -3 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$28. \quad \left[\begin{array}{ccc|c} 1 & 0 & -7 & 10 \\ 0 & 1 & 3 & 25 \end{array} \right]$$

Exercises 29–41, perform row operations on the augmented matrix as far as necessary to determine whether the system is independent, consistent, or inconsistent. (See Examples 6–8.)

$$\begin{array}{rcl} 2x + 2y & = & 0 \\ x + z & = & 2 \\ x + y + z & = & -2 \end{array}$$

$$\begin{array}{rcl} 4x + 2y + 4z & = & 6 \\ x + y + z & = & 1 \\ x + y + 5z & = & 10 \end{array}$$

$$\begin{array}{rcl} x + 4y + 2z & = & -3 \\ x + 2y + z & = & 12 \\ x + y + 4z & = & 3 \end{array}$$

$$\begin{array}{rcl} 30. \quad x + 2y + z & = & 0 \\ & y + 2z & = & 0 \\ & x + y - z & = & 0 \end{array}$$

$$\begin{array}{rcl} 32. \quad x + y + 2z + 3w & = & 1 \\ & 2x + y + 3z + 4w & = & 1 \\ & 3x + y + 4z + 5w & = & 2 \end{array}$$

$$\begin{array}{rcl} 34. \quad 2x + 2y + 2z & = & 6 \\ & 3x - 3y - 4z & = & -1 \\ & x + y + 3z & = & 11 \end{array}$$

Exercises 42–64, use the augmented matrix of the system and use the matrix method to solve the system. (See Examples 2 and 4.)

$$\begin{array}{rcl} 35. \quad x + 2y + 2z & = & 0 \\ & 2x - y + z & = & 3 \\ & x + 2y + 3z & = & 0 \end{array}$$

$$\begin{array}{rcl} 37. \quad x + 2y + 4z & = & 6 \\ & x + 2y + 13z & = & 6 \\ & 2x + 6y + z & = & -10 \end{array}$$

$$\begin{array}{rcl} 39. \quad x + y + z & = & 200 \\ & x - 2y & = & 0 \\ & 2x + 3y + 5z & = & 600 \\ & 2x - y + z & = & 200 \end{array}$$

$$\begin{array}{rcl} 41. \quad x + y + z & = & 5 \\ & 2x + y + z & = & 2 \\ & x - y + z & = & -2 \end{array}$$

$$\begin{array}{rcl} 36. \quad 3x + 7y + 9z & = & 0 \\ & x + 2y + 3z & = & 2 \\ & x + 4y + z & = & 2 \end{array}$$

$$\begin{array}{rcl} 38. \quad x - 2y + 5z & = & -6 \\ & x + 2y + 3z & = & 0 \\ & x + 3y + 2z & = & 5 \end{array}$$

$$\begin{array}{rcl} 40. \quad 2x - y + 2z & = & 3 \\ & -x + 2y - z & = & 0 \\ & 3y - 2z & = & 1 \\ & x + y - z & = & 1 \end{array}$$

$$\begin{array}{rcl} 42. \quad 2x + y + 3z & = & 9 \\ & -x - y + z & = & 1 \\ & 3x - y + z & = & 9 \end{array}$$

Use the Gauss–Jordan method to solve each of the given systems of equations. (See Examples 3–5.)

$$\begin{array}{rcl} 43. \quad x + 2y + z & = & 5 \\ & 2x + y - 3z & = & -2 \\ & 3x - y + 4z & = & -5 \end{array}$$

$$\begin{array}{rcl} 45. \quad x + 3y - 6z & = & 7 \\ & 2x - y + 2z & = & 0 \\ & x + y + 2z & = & -1 \end{array}$$

$$\begin{array}{rcl} 47. \quad x + 2y + 4z & = & 9 \\ & x + y + 13z & = & 6 \\ & -2x + 6y - z & = & -10 \end{array}$$

$$\begin{array}{rcl} 44. \quad 3x - 2y + z & = & 6 \\ & 3x + y - z & = & -4 \\ & -x + 2y - 2z & = & -8 \end{array}$$

$$\begin{array}{rcl} 46. \quad x & = & 1 - y \\ & 2x & = & z \\ & 2z & = & -2 - y \end{array}$$

$$\begin{array}{rcl} 48. \quad x - y + 5z & = & -6 \\ & 3x + 3y - z & = & 10 \\ & x + 2y + 3z & = & 5 \end{array}$$

Solve the system by any method.

$$\begin{array}{rcl} 49. \quad x + 3y + 4z & = & 14 \\ & 2x - 3y + 2z & = & 10 \\ & 3y - y + z & = & 9 \\ & 4x + 2y + 5z & = & 9 \end{array}$$

$$\begin{array}{rcl} 51. \quad 4x + 4y + 12z & = & 28 \\ & 3x - 2y - z & = & 36 \end{array}$$

$$\begin{array}{rcl} 50. \quad x - y & = & 2 \\ & x + y & = & 4 \\ & 2x + 3y & = & 9 \\ & 3x - 2y & = & 6 \end{array}$$

$$\begin{array}{rcl} 52. \quad 3x + y - z & = & 0 \\ & 2x - y + 3z & = & -7 \end{array}$$

$$\begin{array}{rcl} 53. \quad -8x - 9y & = & 11 \\ & 24x + 34y & = & 2 \\ & 16x + 11y & = & -57 \end{array}$$

$$\begin{array}{rcl} 55. \quad x - 2y + z & = & 5 \\ & 2x + y - z & = & 2 \\ & -2x + 4y - 2z & = & 2 \end{array}$$

$$\begin{array}{rcl} 57. \quad x + y - z & = & -20 \\ & 2x - y + z & = & 11 \end{array}$$

$$\begin{array}{rcl} 59. \quad 2x + y + 3z - 2w & = & -6 \\ & 4x + 3y + z - w & = & -2 \\ & x + y + z + w & = & -5 \\ & -2x - 2y - 2z + 2w & = & -10 \end{array}$$

$$\begin{array}{rcl} 60. \quad x + y + z + w & = & -1 \\ & -x + 4y + z - w & = & 0 \\ & x - 2y + z - 2w & = & 11 \\ & -x - 2y + z + 2w & = & -9 \end{array}$$

$$\begin{array}{rcl} 61. \quad x + 2y - z & = & 3 \\ & 3x + y + w & = & 4 \\ & 2x - y + z + w & = & 2 \end{array}$$

$$\begin{array}{rcl} 63. \quad 2x + 3y + z & = & 9 \\ & 4x + y - 3z & = & -7 \\ & 6x + 2y - 4z & = & -8 \end{array}$$

$$\begin{array}{rcl} 54. \quad x + 2y & = & 3 \\ & 2x + 3y & = & 4 \\ & 3x + 4y & = & 5 \\ & 4x + 5y & = & 6 \end{array}$$

$$\begin{array}{rcl} 56. \quad 2x + y & = & 7 \\ & x - y & = & 3 \\ & x + 3y & = & 4 \end{array}$$

$$\begin{array}{rcl} 58. \quad 4x + 3y + z & = & 1 \\ & -2x - y + 2z & = & 0 \end{array}$$

$$\begin{array}{rcl} 62. \quad 3x - 2y - 8z & = & 1 \\ & 9x - 6y - 24z & = & -2 \\ & x - y + z & = & 1 \end{array}$$

$$\begin{array}{rcl} 64. \quad x - 2y - z - 3w & = & -3 \\ & -x + y + z & = & 2 \\ & 4y + 3z - 6w & = & -2 \end{array}$$

Work these exercises:

65. **Population Growth** The population y in year x of the county listed is approximated by:

$$\text{Calhoun County, IL: } x + 20y = 112$$

$$\text{Martin County, TX: } -x + 10y = 38$$

$$\text{Schley County, GA: } y = 5,$$

where $x = 10$ corresponds to the year 2010 and y is in thousands. Solve this system of equations, and interpret your answer. (Data from: U.S. Census Bureau.)

66. **Population Growth** The population y in year x of the greater metropolitan area listed is approximated by the given equation in which $x = 10$ corresponds to the year 2010 and y is in thousands:

$$\text{Portland, OR: } -32x + y = 1906$$

$$\text{Cambridge, MA: } -22x + y = 2026$$

$$\text{Charlotte, NC: } -73x + 2y = 3704.$$

Solve this system of equations, and interpret your answer. (Data from: U.S. Census Bureau.)

Use a graphing calculator to solve these problems.

67. **Concert Tickets** A band concert is attended by x adults, y teenagers, and z preteen children. These numbers satisfy the following equations:

$$x + 1.25y + .25z = 457.5$$

$$x + .6y + .4z = 390$$

$$3.16x + 3.48y + .4z = 1297.2.$$

How many adults, teenagers, and children were present?

68. **Credit Card Debt** The owner of a small business borrows money on three separate credit cards: x dollars on Mastercard, y dollars on Visa, and z dollars on American Express. These amounts satisfy the following equations:

$$1.18x + 1.15y + 1.09z = 11,244.25$$

$$3.54x - .55y + .27z = 3,732.75$$

$$.06x + .05y + .03z = 414.75$$

How much did the owner borrow on each card?

Work these problems.

69. Graph the equations in the given system. Then explain why the graphs show that the system is inconsistent:

$$2x + 3y = 8$$

$$x - y = 4$$

$$5x + y = 7$$

70. Explain why the graphs of the equations in the given system suggest that the system is independent:

$$-x + y = 1$$

$$-2x + y = 0$$

$$x + y = 3$$

71. Find constants a , b , and c such that the points $(2, 3)$, $(-1, 0)$, and $(-2, 2)$ lie on the graph of the equation $y = ax^2 + bx + c$. (Hint: Since $(2, 3)$ is on the graph, we must have $3 = a(2^2) + b(2) + c$; that is, $4a + 2b + c = 3$. Similarly, the other two points lead to two more equations. Solve the resulting system for a , b , and c .)
72. Explain why a system with more variables than equations cannot have a unique solution (that is, be an independent system). (Hint: When you apply the elimination method to such a system, what must happen?)

3. (a) $\begin{bmatrix} 3 & -2 \\ -1 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & 10 \\ 3 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 5 \\ 6 & -17 \end{bmatrix}$

(d) $\begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}$

4. $(-1, 1, 2)$

5. (a) $\begin{bmatrix} 1 & 2 & 11 \\ -4 & 1 & -8 \\ 5 & 1 & 19 \end{bmatrix}$

(b) $4R_1 + R_2 \begin{bmatrix} 1 & 2 & 11 \\ 0 & 9 & 36 \\ 5 & 1 & 19 \end{bmatrix}$

(c) $-5R_1 + R_3 \begin{bmatrix} 1 & 2 & 11 \\ 0 & 9 & 36 \\ 0 & -9 & -36 \end{bmatrix}$

(d) $\frac{1}{9}R_2 \begin{bmatrix} 1 & 2 & 11 \\ 0 & 1 & 4 \\ 0 & -9 & -36 \end{bmatrix}$

(e) $-2R_2 + R_1 \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & -9 & -36 \end{bmatrix}$

(f) $9R_2 + R_3 \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

(g) $(3, 4)$

6. $(1, 2, -1)$

7. $(-5, 3)$

8. (a) No solution

(b) $(0, 0)$

9. (a) $(-50, -26, 7)$

(b) $(55, 16, -14)$

(c) $(-40, -22, 5)$

Checkpoint Answers

1. (a) $(-1, 1)$

(b) $(3, 1, -1)$

2. (a) $\begin{bmatrix} 4 & -2 & 3 & 4 \\ 3 & 5 & 1 & -7 \\ 5 & -1 & 4 & 6 \end{bmatrix}$

(b) $2x - 2y = -2$

$x + y = 4$

$3x + 5y = 8$

6.3 Applications of Systems of Linear Equations

There are no hard and fast rules for solving applied problems, but it is usually best to begin by identifying the unknown quantities and letting each be represented by a variable. Then look at the given data to find one or more relationships among the unknown quantities that lead to equations. If the equations are all linear, use the techniques of the preceding sections to solve the system.

6.3 Exercises

Use systems of equations to work these applied problems. (See Examples 1–5.)

- Lyft Rates** Lyft charges customers a flat fee per ride (which is higher during rush hour traffic) plus charges for each minute and each mile. (Data from: www.lyft.com.) Suppose that, in a certain metropolitan area during rush hour, the flat fee is \$4, the cost per minute is \$.20, and the cost per mile is \$1.40. Let x be the number of minutes and y the number of miles. At the end of a ride, the driver said that you owed \$14 and remarked that the number of minutes was three times the number of miles. Find the number of minutes and the number of miles for this trip.
- Uber Rates** Uber charges customers a flat fee per ride (which is higher during rush hour traffic) plus charges for each minute and each mile. (Data from: www.uber.com.) For a certain trip in Birmingham, Alabama, in 2016, Uber charged a \$3.05 flat fee plus \$0.18 per minute and \$1.20 per mile. Let x be the number of minutes and y the number of miles. At the end of the ride, the driver said that the bill was \$10.85 and remarked that the number of minutes was twice the number of miles. Find the number of minutes and the number of miles for this trip.
- Euro and Pounds** Abigail Henderson has \$1000 budgeted for spending money on an upcoming trip to London and Paris. The European currency is trading at \$1.20 per euro, and British pounds sterling are trading at \$1.40 per pound. She plans to spend more time in France, so she wants to have three times as many euros as pounds. Set up and solve a system of equations to model this problem, and explain what your answer means in practical terms.
- Pesos and Canadian Dollars** Emma Henderson is planning a North American odyssey and has \$1500 budgeted for spending money. It costs \$.60 to buy a peso and \$.75 to buy a Canadian dollar. She expects her adventures in Mexico to take more time, so she wants to have five times as many pesos as Canadian dollars. Set up and solve a system of equations to model this problem, and explain what your answer means in practical terms.
- Stock Portfolio** You invested a total of \$25,000 in Coca-Cola Company and Intel Corporation. You purchased Coca-Cola at \$45 per share and Intel at \$32 per share. If Coca-Cola doubles in value and Intel stock goes up by 50%, your portfolio will be worth \$42,000. How many shares of each stock do you own?
- Bond Portfolio** You purchased a total of \$10,000 in bond funds. The JP Morgan Emerging Markets bond fund costs \$100 per share, and the Fidelity Conservative Income bond fund costs \$10 per share. If the emerging markets fund goes up in value by 5%, and the conservative bond fund increases by 10%, your portfolio will be worth \$10,600. How many shares of each fund do you own?
- Basketball Statistics** In a championship basketball game, the center scored a total of 31 points. Let x be the number of free-throw baskets made (worth 1 point each), y the number of regular baskets made (2 points each), and z the number of baskets made from behind the three-point line (3 points each). The player made a total of 12 baskets from the floor (2-point and 3-point shots) and 4 baskets from the free-throw line.
 - Set up a system of three equations to model this situation.
 - Solve the system and interpret your answer.
- Baseball Statistics** In the final game of a baseball series that went to extra innings, the most valuable player (MVP) rounded a total of 12 bases. Let x be the number of singles, y the number of doubles, z the number of triples, and w the number of home runs hit. The MVP had 1 single and 4 hits for extra bases, including 1 home run.
 - Set up a system of four equations to model this situation.
 - Solve the system and interpret your answer.
- Truck Rentals** The rent-a-truck company of Example 1 learns that each van now costs \$25,000, each small truck costs \$30,000, and each large truck costs \$40,000. The company still needs twice as many vans as small trucks and wants to spend \$5 million, but it decides to buy only 175 vehicles. How many of each kind should it buy?
- Finance** Suppose that the investor in Example 2 finds that her annual return on the international stock fund will be only 4%. Now how much should she put in each investment?
- Livestock Dietary Requirements** To meet consumer demand, the animal feed in Example 3 now supplies only 2400 units of fat. How many units of corn, soybeans, and cottonseed are now needed?
- Delivery Logistics** Suppose that the wholesaler in Example 3 finds a way to pack the EZ model in an 8-cubic-foot box. Now how many of each model should a fully loaded van carry?
- Dry Cleaning** Mario took clothes to the cleaners three times last month. First, he brought 3 shirts and 1 pair of slacks and paid \$10.96. Then he brought 7 shirts, 2 pairs of slacks, and a sport coat and paid \$30.40. Finally, he brought 4 shirts and 1 sports coat and paid \$14.45. How much was he charged for each shirt, each pair of slacks, and each sports coat?
- Online Retailing** Shipping charges at an online bookstore are \$4 for one book, \$6 for two books, and \$7 for three or more books. Last week, there were 6400 orders of five or fewer books, and total shipping charges for these orders were \$33,600. The number of shipments with \$7 charges was 1000 less than the number with \$6 charges. How many shipments were made in each category (one book, two books, three or more books)?
- Concert Tickets** Tickets to a band concert cost \$5 for adults, \$3 for teenagers, and \$2 for preteens. There were 570 people at the concert, and total ticket receipts were \$1950. Three-fourths as many teenagers as preteens attended. How many adults, teenagers, and preteens attended?
- Minor League Baseball** A minor league baseball team had 7000 fans in attendance, and total ticket receipts were \$26,400.

- Box seats cost \$6, grandstand seats cost \$4, and bleacher seats cost \$2. If the number of tickets sold for box seats was one-third the number of bleacher seats sold, how many tickets of each type were sold?
17. **Finance** An investment firm recommends that a client invest in bonds rated AAA, A, and B. The average yield on AAA bonds is 6%, on A bonds 7%, and on B bonds 10%. The client wants to invest twice as much in AAA bonds as in B bonds. How much should be invested in each type of bond if the total investment is \$25,000, and the investor wants an annual return of \$1,100 on the three investments?
18. **Finance** An investor wants to invest \$30,000 in corporate bonds that are rated AAA, A, and B. The lower rated ones pay higher interest, but pose a higher risk as well. The average yield is 5% on AAA bonds, 6% on A bonds, and 10% on B bonds. Being conservative, the investor wants to have twice as much in AAA bonds as in B bonds. How much should she invest in each type of bond to have an interest income of \$2000?
19. **Finance** An investor plans to invest \$70,000 in a mutual fund, corporate bonds, and a fast-food franchise. She plans to put twice as much in bonds as in the mutual fund. On the basis of past performance, she expects the mutual fund to pay a 2% dividend, the bonds 6%, and the franchise 10%. She would like a dividend income of \$4800. How much should she put in each of three investments?
20. **Finance** Makayla borrows \$10,000. Some is from her friend at 8% annual interest, twice as much as that from her bank at 9%, and the remainder from her insurance company at 5%. She pays a total of \$830 in interest for the first year. How much did she borrow from each source?
21. **Trail Mix Recipe** Pretzels cost \$3 per pound, dried fruit \$4 per pound, and nuts \$8 per pound. How many pounds of each should be used to produce 140 pounds of trail mix costing \$6 per pound in which there are twice as many pretzels (by weight) as dried fruit?
22. **Auto Manufacturing** An auto manufacturer sends cars from two plants, I and II, to dealerships A and B, located in a midwestern city. Plant I has a total of 28 cars to send, and plant II has 8. Dealer A needs 20 cars, and dealer B needs 16. Transportation costs based on the distance of each dealership from each plant are \$220 from I to A, \$300 from I to B, \$400 from II to A, and \$180 from II to B. The manufacturer wants to limit transportation costs to \$10,640. How many cars should be sent from each plant to each of the two dealerships?
23. **Animal Feed** An animal breeder can buy four types of tiger food. Each case of Brand A contains 25 units of fiber, 30 units of protein, and 30 units of fat. Each case of Brand B contains 50 units of fiber, 30 units of protein, and 20 units of fat. Each case of Brand C contains 75 units of fiber, 30 units of protein, and 20 units of fat. Each case of Brand D contains 100 units of fiber, 60 units of protein, and 30 units of fat. How many cases of each brand should the breeder mix together to obtain a food that provides 1200 units of fiber, 600 units of protein, and 400 units of fat?
24. **Physical Science** The stopping distance for a car traveling 25 mph is 61.7 feet, and for a car traveling 35 mph it is 106 feet. The stopping distance in feet can be described by the equation

$y = ax^2 + bx$, where x is the speed in mph. (Data from: National Traffic Safety Institute.)

- Find the values of a and b .
- Use your answers from part (a) to find the stopping distance for a car traveling 55 mph.

Work these exercises. (See Example 4.)

25. **Female Population** The table shows Census Bureau projections for the female population of the United States (in millions).

Year	2015	2035	2040
Female Population	163	187	192

- Find a quadratic function $f(x) = ax^2 + bx + c$ that gives the female population (in millions) in year x , where $x = 0$ corresponds to the year 2000.
- Estimate the female population in the year 2020.

26. **Male Population** The table shows Census Bureau projections for the male population of the United States (in millions).

Year	2015	2035	2040
Male Population	158	183	188

- Find a quadratic function $f(x) = ax^2 + bx + c$ that gives the male population (in millions) in year x , where $x = 0$ corresponds to the year 2000.
- Estimate the male population in the year 2020.

27. **Fuel Consumption** At a pottery factory, fuel consumption for heating the kilns varies with the size of the order being fired. In the past, the company recorded the figures in the table.

x = Number of Platters	y = Fuel Cost per Platter
6	\$2.80
8	2.48
10	2.24

- Find an equation of the form $y = ax^2 + bx + c$ whose graph contains the three points corresponding to the data in the table.
- How many platters should be fired at one time in order to minimize the fuel cost per platter? What is the minimum fuel cost per platter?

28. **Alzheimer's Disease** The number of Alzheimer's cases in people 85 and older was 2 million in 2004 and is projected to be 3 million in 2025 and 8 million in 2050. (Data from: Alzheimer's Association.)

- Let $x = 0$ correspond to 2000. Find a quadratic function that models the given data.
- How many people 85 or older will have Alzheimer's disease in 2020 and in 2034?
- In the year you turn 85, how many people your age or older are expected to have Alzheimer's disease?

 A graphing calculator or other technology is recommended for the following exercises.

29. **Health** The table shows the calories, sodium, and protein in one cup of various kinds of soup.

	Progresso™ Hearty Chicken Rotini	Healthy Choice™ Hearty Chicken	Campbell's™ Chunky Chicken Noodle
Calories	100	130	110
Sodium (mg)	960	480	890
Protein (g)	7	9	8

How many cups of each kind of soup should be mixed together to produce 10 servings of soup, each of which provides 171 calories, 1158 milligrams of sodium, and 12.1 grams of protein? What is the serving size (in cups)? (*Hint:* In 10 servings, there must be 1710 calories, 11,580 milligrams of sodium, and 121 grams of protein.)

30. **Health** The table shows the calories, sodium, and fat in 1 ounce of various snack foods (all produced by Planters™):

	Sweet N' Crunchy Peanuts	Dry Roasted Honey Peanuts	Kettle Roasted Honey BBQ Peanuts
Calories	140	160	180
Sodium (mg)	20	110	55
Fat (g)	8	13	15

How many ounces of each kind of snack should be combined to produce 10 servings, each of which provides 284 calories, 93 milligrams of sodium, and 20.6 grams of fat? What is the serving size?

Checkpoint Answers

1. $x + y + z = 150$
 $4x + 5y + 7z = 400$
 $x - 3y = 0$

2. (a)
$$\begin{bmatrix} 1 & 1 & 1 & 1 & 100,000 \\ .025 & .035 & .05 & .06 & 4000 \\ 0 & 0 & 1 & -.25 & 0 \\ 1 & 1 & 0 & 0 & 60,000 \end{bmatrix}$$

- (b) Many sequences are possible, including this one:

Replace R_2 by $-.025R_1 + R_2$;

replace R_4 by $-R_1 + R_4$;

replace R_2 by $\frac{1}{.01}R_2$;

replace R_4 by $R_3 + R_4$;

replace R_4 by $\frac{-1}{1.25}R_4$.

(c)
$$\begin{bmatrix} 1 & 1 & 1 & 1 & 100,000 \\ 0 & 1 & 2.5 & 3.5 & 150,000 \\ 0 & 0 & 1 & -.25 & 0 \\ 0 & 0 & 0 & 1 & 32,000 \end{bmatrix}$$

3. (a)
$$\begin{bmatrix} 10 & 20 & 30 & 1800 \\ 30 & 20 & 40 & 2800 \\ 20 & 40 & 25 & 2200 \end{bmatrix}$$

- (b) Many sequences are possible, including this one:

Replace R_1 by $\frac{1}{10}R_1$;

replace R_2 by $\frac{1}{10}R_2$;

replace R_3 by $\frac{1}{5}R_3$;

replace R_2 by $-3R_1 + R_2$;

replace R_3 by $-4R_1 + R_3$;

replace R_2 by $-\frac{1}{4}R_2$;

replace R_3 by $-\frac{1}{7}R_3$.

(c)
$$\begin{bmatrix} 1 & 2 & 3 & 180 \\ 0 & 1 & \frac{5}{4} & 65 \\ 0 & 0 & 1 & 40 \end{bmatrix}$$

6.4 Basic Matrix Operations

Until now, we have used matrices only as a convenient shorthand to solve systems of equations. However, matrices are also important in the fields of management, natural science, engineering, and social science as a way to organize data, as Example 1 demonstrates.

EXAMPLE 1

Furniture Manufacturing The EZ Life Company manufactures sofas and armchairs in three models: A, B, and C. The company has regional warehouses in New York, Chicago, and San Francisco. In its August shipment, the company sends 10 model A sofas, 12 model B sofas, 5 model C sofas, 15 model A chairs, 20 model B chairs, and 8 model C chairs to each warehouse.

6.4 Exercises

Find the size of each of the given matrices. Identify any square, column, or row matrices. Give the negative (additive inverse) of each matrix. (See Examples 2 and 9.)

1. $\begin{bmatrix} 7 & -8 & 4 \\ 0 & 13 & 9 \end{bmatrix}$

2. $\begin{bmatrix} -7 & 23 \\ 5 & -6 \end{bmatrix}$

3. $\begin{bmatrix} -3 & 0 & 11 \\ 1 & \frac{1}{4} & -7 \\ 5 & -3 & 9 \end{bmatrix}$

4. $\begin{bmatrix} 6 & -4 & \frac{2}{3} & 12 & 2 \end{bmatrix}$

5. $\begin{bmatrix} 7 \\ 11 \end{bmatrix}$

6. $[-5]$

7. If A is a 3×3 matrix and $A + B = A$, what do you know about B ?

8. If C is a 3×3 matrix and D is a 3×4 matrix, then $C + D$ is _____.

Perform the indicated operations where possible. (See Examples 3–7.)

9. $\begin{bmatrix} 1 & 2 & 7 & -1 \\ 8 & 0 & 2 & -4 \end{bmatrix} \div \begin{bmatrix} -8 & 12 & -5 & 5 \\ -2 & -3 & 0 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 7 \\ 2 & -3 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 8 \\ 6 & 8 \\ -1 & 9 \end{bmatrix}$

11. $\begin{bmatrix} -1 & -5 & 9 \\ 2 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 4 & -7 \\ 1 & -1 & 2 \end{bmatrix}$

12. $\begin{bmatrix} 2 & 4 \\ -8 & 2 \end{bmatrix} + \begin{bmatrix} 9 & -5 \\ 8 & 5 \end{bmatrix}$

13. $\begin{bmatrix} 0 & -2 \\ 1 & 9 \\ -5 & -9 \end{bmatrix} - \begin{bmatrix} 8 & 6 \\ 9 & 17 \\ 3 & -1 \end{bmatrix}$

14. $\begin{bmatrix} -3 & -2 & 5 \\ 3 & 9 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -3 & 6 & 8 \end{bmatrix}$

15. $\begin{bmatrix} 9 & 1 \\ 0 & -3 \\ 4 & 10 \end{bmatrix} - \begin{bmatrix} 1 & 9 & -4 \\ -1 & 1 & 0 \end{bmatrix}$

16. $\begin{bmatrix} 3 & -8 & 0 \\ 1 & 5 & -7 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ 9 & -4 \end{bmatrix}$

Let $A = \begin{bmatrix} -2 & 0 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2 \\ 4 & -6 \end{bmatrix}$. Find each of the following. (See Examples 8 and 9.)

17. $2A$

18. $-3B$

19. $-4B$

20. $5A$

21. $-4A + 5B$

22. $3A - 10B$

Let $A = \begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ 0 & 5 \end{bmatrix}$. Find a matrix X satisfying the given equation.

23. $2X = 2A + 3B$

24. $3X = A - 3B$

Using matrices

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, P = \begin{bmatrix} m & n \\ p & q \end{bmatrix}, T = \begin{bmatrix} r & s \\ t & u \end{bmatrix}, \text{ and } X = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

verify that the statements in Exercises 25–29 are true.

25. $X + T$ is a 2×2 matrix.

26. $X + T = T + X$ (commutative property of addition of matrices).

27. $X + (T + P) = (X + T) + P$ (associative property of addition of matrices).

28. $X + (-X) = O$ (inverse property of addition of matrices).

29. $P + O = P$ (identity property of addition of matrices).

30. Which of the preceding properties are valid for matrices that are not square?

Work the following exercises. (See Example 1.)

31. **Concessions and Merchandise** When ticket holders fail to attend, major league sports teams lose the money those fans would have spent on refreshments and merchandise. The percentage of fans who don't show up is 16% in basketball and hockey, 20% in football, and 18% in baseball. The lost revenue per fan is \$18.20 in basketball, \$18.25 in hockey, \$19 in football, and \$15.40 in baseball. The total annual lost revenue is \$22.7 million in basketball, \$35.8 million in hockey, \$51.9 million in football, and \$96.3 million in baseball. Express this information in matrix form; specify what the rows and columns represent. (Data from: American Demographics.)

32. **Earnings for College Graduates** In 2015, median weekly earnings were \$493 for workers without a high school diploma, \$678 for high school graduates, and \$1137 for those who had earned a bachelor's degree. The unemployment rate was 2.5% for college graduates, 5.4% for high school graduates, and 8.0% for those with less than a high school diploma. Write this information as a 3×2 matrix, labeling rows and columns. (Data from: Bureau of Labor Statistics.)

33. **Organ Transplants** The shortage of organs for transplants is a continuing problem in the United States. In 2016, there were 4154 candidates waiting for a heart transplant, 1470 for a lung transplant, 14,794 for a liver transplant, and 100,467 for a kidney transplant. Corresponding figures for 2010 were 2874, 15,394, and 83,919, respectively. Express this information as a matrix, labeling rows and columns. (Data from: Organ Procurement and Transplant Network.)

34. **Household Debt** In 2015, only 38% of American households had credit card debt, but for households who carried credit card

the average amount of debt per household was \$15,762. Averages for households with other types of debt included \$2,141 in auto loans, \$48,172 in student loan, and \$168,614 in mortgage debt. The corresponding total figures for all consumers in the United States are \$733 billion, \$1060 billion, \$1230 billion, and \$8250 billion, respectively. Write this information in a 4×2 matrix, labeling rows and columns. (Data from: Federal Reserve Bank of New York.)

4. These exercises.

5. **Convenience Stores** There are three convenience stores in a neighborhood. This week, Store I sold 88 loaves of bread, 48 quarts of milk, 16 jars of peanut butter, and 112 pounds of cold cuts. Store II sold 105 loaves of bread, 72 quarts of milk, 21 jars of peanut butter, and 147 pounds of cold cuts. Store III sold 60 loaves of bread, 40 quarts of milk, no peanut butter, and 50 pounds of cold cuts.

(a) Use a 3×4 matrix to express the sales information for the three stores.

(b) During the following week, sales on these products at Store I increased by 25%, sales at Store II increased by one-third, and sales at Store III increased by 10%. Write the sales matrix for that week.

(c) Write a matrix that represents total sales over the two-week period.

6. **Female and Male Graduates** The following table gives educational attainment as a percent of the U.S. population 25 years and older in various years. (Data from: U.S. Census Bureau.)

Year	MALE		FEMALE	
	Four Years of High School or More	Four Years of College or More	Four Years of High School or More	Four Years of College or More
1940	22.7%	5.5%	26.3%	3.8%
1970	55.0%	14.1%	55.4%	8.2%
2000	84.2%	27.8%	84.0%	23.6%
2014	87.7%	31.9%	88.9%	32.0%

- (a) Write a 2×4 matrix for the educational attainment of males.
 (b) Write a 2×4 matrix for the educational attainment of females.
 (c) Use the matrices from parts (a) and (b) to write a matrix showing how much more (or less) males have reached educational attainment than females.

Checkpoint Answers

1. $\begin{bmatrix} 10 & 15 \\ 12 & 20 \\ 5 & 8 \end{bmatrix}$

2. (a) 2×4

(b) 3×3

3. (a) $\begin{bmatrix} 2 & 5 & -8 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 5 \\ -8 \\ 4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 5 \\ -8 & 4 \end{bmatrix}$ or $\begin{bmatrix} 2 & -8 \\ 5 & 4 \end{bmatrix}$
 (Other answers are possible.)

4. (a) $\begin{bmatrix} 1 & 7 & 7 \\ 13 & -5 & 9 \end{bmatrix}$

(b) Not possible

(c) $\begin{bmatrix} 0 & 6 & 2 \end{bmatrix}$

5. (a) 139

(b) 92

(c) 100

6. $\begin{bmatrix} 62 & 13 \\ 24 & -51 \end{bmatrix}$

7. (a) $\begin{bmatrix} -4 & 1 \\ -4 & 2 \end{bmatrix}$

(b) Not possible

(c) $\begin{bmatrix} -1 & -4 & 4 \end{bmatrix}$

8. (a) $\begin{bmatrix} -12 & 6 \\ -3 & -15 \end{bmatrix}$

(b) $\begin{bmatrix} 8 & 16 & 28 \\ 32 & 8 & 4 \\ 20 & 28 & 12 \end{bmatrix}$

9. (a) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 6 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 6 & 3 \end{bmatrix}$

(c) $A + (-B) = A - B$.

6.5 Matrix Products and Inverses

To understand the reasoning behind the definition of matrix multiplication, look again at the EZ Life Company. Suppose sofas and chairs of the same model are often sold as sets, with matrix W showing the number of each model set in each warehouse:

$$\begin{array}{l} \text{New York} \\ \text{Chicago} \\ \text{San Francisco} \end{array} \begin{bmatrix} A & B & C \\ 10 & 7 & 3 \\ 5 & 9 & 6 \\ 4 & 8 & 2 \end{bmatrix} = W.$$

The best way to find matrix inverses on a graphing calculator is illustrated in the next example.

EXAMPLE 6 Use a graphing calculator to find the inverse of the following matrices (if they have inverses):

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -2 & -1 \\ 5 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$$

Solution Enter matrix A into the calculator [Figure 6.25(a)]. Then use the x^{-1} key to find the inverse matrix, as in Figure 6.25(b). (Using $\wedge$ and -1 for the inverse results in an error message on most calculators.) Note that Figure 6.25(b) agrees with the answer we found working by hand in Example 7.

Checkpoint 9

Use a graphing calculator to find the inverses of these matrices (if they exist):

(a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & -1 & 0 \\ 5 & 1 & 3 \end{bmatrix}$

(b) $B = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$

[A]

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & -2 & -1 \\ 5 & 0 & 0 \end{bmatrix}$$

(a)

[A]⁻¹

$$\begin{bmatrix} 0 & 0 & .2 \\ -.5 & -.5 & .3 \\ 1 & 0 & -.2 \end{bmatrix}$$

(b)

Figure 6.25

Now enter matrix B into the calculator and use the x^{-1} key. The result is an error message (Figure 6.26), which indicates that the matrix is singular; it does not have an inverse. If you were working by hand, you would have found that the appropriate system of equations has no solution.)

ERR: SINGULAR MAT
1:Quit
2:Goto

Figure 6.26

TECHNOLOGY TIP

Because of round-off error, a graphing calculator may sometimes display an "inverse" for a matrix that doesn't actually have one. So always verify your results by multiplying A and A^{-1} . If the product is not the identity matrix, then A does not have an inverse.

6.5 Exercises

In Exercises 1–6, the sizes of two matrices A and B are given. Find the sizes of the product AB and the product BA whenever these products exist. (See Example 1.)

1. A is 2×2 and B is 2×2 .
2. A is 3×3 and B is 3×2 .
3. A is 3×5 and B is 5×3 .
4. A is 4×3 and B is 3×6 .
5. A is 4×2 and B is 3×4 .
6. A is 7×3 and B is 2×7 .
7. To find the product matrix AB , the number of _____ of A must be the same as the number of _____ of B .
8. The product matrix AB has the same number of _____ as A and the same number of _____ as B .

Find each of the following matrix products, if they exist. (See Examples 2 and 3.)

9. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

10. $\begin{bmatrix} -2 & 5 \\ 7 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix}$

11. $\begin{bmatrix} 2 & 2 & -1 \\ 5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ -1 & 5 \\ 0 & 2 \end{bmatrix}$

12. $\begin{bmatrix} -9 & 3 & 1 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

13. $\begin{bmatrix} -4 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -5 \end{bmatrix}$

15. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -5 & 7 \\ -2 & 1 & 6 \\ 0 & -3 & 4 \end{bmatrix}$

16. $\begin{bmatrix} -8 & 9 \\ 3 & -4 \\ -1 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 4 \\ 7 & 0 \\ 1 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 & -1 & 5 \\ 0 & 1 & 0 & -1 \\ 4 & 2 & 5 & -4 \end{bmatrix}$$

Exercises 19–21, use the matrices

$$A = \begin{bmatrix} -3 & -9 \\ 2 & 6 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix}$$

19. Show that $AB \neq BA$. Hence, matrix multiplication is not commutative.

20. Show that $(A + B)^2 = A^2 + 2AB + B^2$.

21. Show that $(A + B)(A - B) \neq A^2 - B^2$.

22. Show that $D^2 = D$, where

$$D = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Given matrices

$$P = \begin{bmatrix} m & n \\ p & q \end{bmatrix}, \quad X = \begin{bmatrix} x & y \\ z & w \end{bmatrix}, \quad \text{and} \quad T = \begin{bmatrix} r & s \\ t & u \end{bmatrix}$$

verify that the statements in Exercises 23–26 are true.

23. $P(XT) = P(XT)$ (associative property of multiplication of matrices)

24. $P(X + T) = PX + PT$ (distributive property)

25. $k(X + T) = kX + kT$ for any real number k

26. $(k + h)P = kP + hP$ for any real numbers k and h

Determine whether the given matrices are inverses of each other by computing their product. (See Example 5.)

$$27. \begin{bmatrix} 5 & 2 \\ 3 & -1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$$

$$28. \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \text{ and } \begin{bmatrix} 5 & 0 \\ -7 & 3 \end{bmatrix}$$

$$29. \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

$$30. \begin{bmatrix} 3 & 5 \\ 7 & 9 \end{bmatrix} \text{ and } \begin{bmatrix} -\frac{9}{8} & \frac{5}{8} \\ \frac{7}{8} & -\frac{5}{8} \end{bmatrix}$$

$$31. \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1.5 & .5 & -1.5 \\ -1 & 0 & 1 \\ .5 & -.5 & .5 \end{bmatrix}$$

$$32. \begin{bmatrix} 2 & 5 & 4 \\ 1 & 4 & 3 \\ 1 & 3 & 2 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 2 & 1 \\ -5 & 8 & 2 \\ 7 & -11 & -3 \end{bmatrix}$$

Find the inverse, if it exists, for each of the given matrices. (See Examples 6 and 7.)

$$33. \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

$$34. \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$35. \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

$$36. \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$37. \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

$$38. \begin{bmatrix} -3 & -5 \\ 6 & 10 \end{bmatrix}$$

$$39. \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$40. \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$41. \begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & 3 \\ 2 & -3 & 4 \end{bmatrix}$$

$$42. \begin{bmatrix} 1 & 4 & 3 \\ 1 & -3 & -2 \\ 2 & 5 & 4 \end{bmatrix}$$

$$43. \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \\ -2 & 3 & -2 \end{bmatrix}$$

$$44. \begin{bmatrix} 1 & 0 & -5 \\ 4 & -7 & 3 \\ 3 & -7 & 8 \end{bmatrix}$$

Use a graphing calculator to find the inverse of each matrix. (See Example 8.)

$$45. \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 0 & 1 & -1 \end{bmatrix}$$

$$46. \begin{bmatrix} 2 & 2 & -4 \\ 2 & 6 & 0 \\ -3 & -3 & 5 \end{bmatrix}$$

$$47. \begin{bmatrix} 1 & 0 & -2 & 0 \\ -2 & 1 & 2 & 2 \\ 3 & -1 & -2 & -3 \\ 0 & 1 & 4 & 1 \end{bmatrix}$$

$$48. \begin{bmatrix} 1 & 1 & 0 & 2 \\ 2 & -1 & 1 & -1 \\ 3 & 3 & 2 & -2 \\ 1 & 2 & 1 & 0 \end{bmatrix}$$

Work these exercises. (See Example 4.)

49. **Grocery Delivery** Several competing companies provide grocery home delivery services. You and your roommate agree to order together and are deciding between two local delivery companies, Timesaver and Wheels. Matrix A shows the number of frozen pizza and quarts of ice cream that you each want to order, and matrix B shows the prices for each company.

$$\begin{array}{cc} & \begin{array}{cc} \text{Pizza} & \text{Ice cream} \end{array} \\ \begin{array}{c} \text{You} \\ \text{Roommate} \end{array} & \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix} = A \end{array}$$

$$\begin{array}{cc} & \begin{array}{cc} \text{Timesaver} & \text{Wheels} \end{array} \\ \begin{array}{c} \text{Pizza} \\ \text{Ice cream} \end{array} & \begin{bmatrix} \$8 & \$7 \\ \$4 & \$5 \end{bmatrix} = B \end{array}$$

(a) Find the matrix product AB .

(b) What do the rows of AB represent?

(c) What do the columns of AB represent?

(d) What would be the total cost (for you and your roommate) if you order from Timesaver? From Wheels?

50. **Bulk Shipments** Boxed.com is a company that ships grocery items in bulk. As the purchasing manager for a start-up, you decide to compare Boxed.com prices with your current supplier for an order of Charmin toilet paper (30 count ultra soft) and Bounty paper towels (12 count double rolls). Matrix A shows the quantities that the production and sales departments need

to order, and matrix B shows the prices for each supplier. (Data from: www.boxed.com.)

$$\begin{array}{l} \text{Production} \\ \text{Sales} \end{array} \begin{bmatrix} \text{Charmin} & \text{Bounty} \\ 15 & 25 \\ 60 & 10 \end{bmatrix} = A$$

$$\begin{array}{l} \text{Charmin} \\ \text{Bounty} \end{array} \begin{bmatrix} \text{Boxed.com} & \text{Current} \\ \$23 & \$24 \\ \$20 & \$18 \end{bmatrix} = B$$

- Find the matrix product AB .
- What do the rows of AB represent?
- What do the columns of AB represent?
- What would the total cost be (for both departments) if you order from Boxed.com? From your current supplier?

51. **Food Truck** A restaurant has a main location and a traveling food truck. The first matrix A shows the number of managers and associates employed. The second matrix B shows the average annual cost of salary and benefits (in thousands of dollars).

$$\begin{array}{l} \text{Restaurant} \\ \text{Food Truck} \end{array} \begin{bmatrix} \text{Managers} & \text{Associates} \\ 5 & 25 \\ 1 & 4 \end{bmatrix} = A$$

$$\begin{array}{l} \text{Managers} \\ \text{Associates} \end{array} \begin{bmatrix} \text{Salary} & \text{Benefits} \\ 41 & 6 \\ 20 & 2 \end{bmatrix} = B$$

- Find the matrix product AB .
- Explain what AB represents.
- According to matrix AB , what is the total cost of salaries for all employees (managers and associates) at the restaurant? The food truck? What is the total cost of benefits for all employees at the restaurant? The food truck?

52. **Teacher Salaries** The average annual wage (in thousands of dollars) for teachers in the United States at various grade levels is shown in matrix W . (Data from: Bureau of Labor Statistics.) A small school district has both city schools and county schools; matrix N shows the number of teachers in each.

$$\begin{array}{l} \text{Preschool and Kindergarten} \\ \text{Elementary and Middle School} \\ \text{Secondary School} \end{array} \begin{bmatrix} \text{Wage (thousands)} \\ 34 \\ 61 \\ 51 \end{bmatrix} = W$$

$$\begin{array}{l} \text{City} \\ \text{County} \end{array} \begin{bmatrix} \text{Preschool and Kindergarten} & \text{Elementary and Middle School} & \text{Secondary School} \\ 25 & 60 & 40 \\ 5 & 12 & 10 \end{bmatrix} = N$$

- Find the matrix product NW .
 - Explain what NW represents.
 - According to matrix NW , what is the total cost of teacher wages in city schools? In county schools? In the entire school district?
53. **Gourmet Pops** Steel City Pops sells fruity, creamy, and cookie pops at locations in Texas, Kentucky, and Alabama (www.steelcitypops.com). Suppose that, for a special event, sales (in hundreds of pops) and prices (in dollars per hundred) were as shown in the following matrices.

$$\begin{array}{l} \text{Fruity} \\ \text{Creamy} \\ \text{Cookie} \end{array} \begin{bmatrix} \text{Price} \\ 250 \\ 300 \\ 350 \end{bmatrix} = P$$

$$\begin{array}{l} \text{Texas} \\ \text{Kentucky} \\ \text{Alabama} \end{array} \begin{bmatrix} \text{Fruity} & \text{Creamy} & \text{Cookie} \\ 8 & 7 & 2 \\ 4 & 5 & 2 \\ 6 & 9 & 4 \end{bmatrix} = S$$

- Is it possible to find the matrix product PS ? If so, find it and explain its meaning.
- Is it possible to find the matrix product SP ? If so, find it and explain its meaning.
- What was the total revenue from all locations for this special event?

54. **Franchise Costs** According to Franchise Business Review, the minimum investments (in thousands) to launch certain franchises are listed in matrix M below. The numbers of franchises that your investment group would like to start this year in Pennsylvania and Indiana are shown in matrix N .

$$\begin{array}{l} \text{Deli Delicious} \\ \text{Taziki's Mediterranean Cafe} \\ \text{Your Pie (brick oven pizza)} \end{array} \begin{bmatrix} \text{Cost} \\ 150 \\ 313 \\ 400 \end{bmatrix} = M$$

$$\begin{array}{l} \text{Indiana} \\ \text{Pennsylvania} \end{array} \begin{bmatrix} \text{Deli} & \text{Taziki's} & \text{Pie} \\ 3 & 0 & 5 \\ 2 & 7 & 1 \end{bmatrix} = N$$

- Is it possible to find the product MN ? If so, find it and explain the meaning of each entry.
- Is it possible to find the product NM ? If so, find it and explain the meaning of each entry.
- What is the total dollar amount that your group must invest for all locations?

Use a graphing calculator for Exercises 55 and 56.

55. **Birth and Death Rates** Population estimates and projections (in millions) for the BRIC countries (Brazil, Russia, India, China) and birth and death rates per million people for these countries are shown in the tables. (Data from: United Nations Population Fund.)

	2010	2020
Brazil	195	210
Russia	143	141
India	1225	1387
China	1341	1388

	Births	Deaths
Brazil	.016	.006
Russia	.011	.014
India	.023	.008
China	.013	.007

- (a) Write the data in the first table as a 2×4 matrix A .
 (b) Write the data in the second table as a 4×2 matrix B .
 (c) Find the matrix product AB .
 (d) Explain what AB represents.
 (e) According to matrix AB , what was the total number of deaths in the BRIC countries in 2010? What total number of births is projected for 2020?



- **Birth and Death Rates** Population estimates and projections (in millions) for several regions of the world and the average birth and death rates per million people for those regions are shown in the tables below. (Data from: United Nations Population Fund.)

	Asia	Europe	Latin America	North America
2010	4164	738	590	345
2020	4566	744	652	374

	Births	Deaths
Asia	.019	.007
Europe	.011	.011
Latin America	.019	.006
North America	.024	.008

- (a) Write the information in the first table as a 2×4 matrix A .
 (b) Write the information in the second table as a 4×2 matrix B .
 (c) Find the matrix product AB .
 (d) Explain what AB represents.
 (e) According to matrix AB , what was the total number of births in these four regions combined in 2010? What total number of deaths is projected for 2020?

Checkpoint Answers

- \$27,100
- (a) No (b) Yes; 2×6
- $CD = \begin{bmatrix} 19 & -2 \\ -13 & -12 \end{bmatrix}$
- (a) 2×3 (b) Not possible (c) 3×3
- (a) A = brand/vitamin; B = vitamin/cost.
 (b) AB = brand/cost.
 (c) Not meaningful, although the product BA can be found.
- $IA = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} = A$, and $AI = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix} = A$.
- No, because $AB \neq I$.
- RREF $[B]$ is matrix (6) in the example.
- (a) No inverse
 (b) Use the FRAC key (if you have one) to simplify the answer, which is

$$B^{-1} = \begin{bmatrix} 4/11 & -3/11 \\ 1/11 & 2/11 \end{bmatrix}$$

6.6 Applications of Matrices

This section gives a variety of applications of matrices.

Solving Systems with Matrices

Consider this system of linear equations:

$$2x - 3y = 4$$

$$x + 5y = 2.$$

Let

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \text{and} \quad B = \begin{bmatrix} 4 \\ 2 \end{bmatrix}.$$

Solution From the diagram, we see that there are two pathways connecting building 1 (B1) to building 2 (B2) without passing through buildings 3, 4, or 5. This information is entered in row one, column two, and again in row two, column one of matrix A :

$$\begin{array}{c} \text{B1} \text{ B2} \text{ B3} \text{ B4} \text{ B5} \\ \text{B1} \begin{bmatrix} 0 & 2 & 1 & 1 & 2 \end{bmatrix} \\ \text{B2} \begin{bmatrix} 2 & 0 & 1 & 0 & 1 \end{bmatrix} \\ \text{B3} \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \end{bmatrix} \\ \text{B4} \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \end{bmatrix} \\ \text{B5} \begin{bmatrix} 2 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array} = A.$$

Note that there are no pathways connecting each building to itself, so the main diagonal of the matrix from upper left to lower right consists entirely of zeros.

- (b) How many ways are there to go from building 1 to building 3 by going through exactly one other building?

Solution This can happen by going from building 1 to building 2 two ways, and then from building 2 to building 3. Another way is to go from building 1 to building 4 and then from building 4 to building 3. Thus, there are three total ways to go from building 1 to building 3 by going through exactly one other building.

- (c) Given any pair of buildings, how many ways are there to go between them by going through exactly one other building?

Solution The matrix A^2 shows the number of ways to travel between any two buildings by going through exactly one other building.

$$A^2 = \begin{bmatrix} 10 & 3 & 3 & 3 & 3 \\ 3 & 6 & 2 & 4 & 4 \\ 3 & 2 & 3 & 1 & 4 \\ 3 & 4 & 1 & 3 & 2 \\ 3 & 4 & 4 & 2 & 6 \end{bmatrix}$$

We ignore the values down the main diagonal for travel from one building into itself. We notice that the entry in row one, column three has a value of 3. This coincides with what we learned in part (b)—that there are three ways to go from building 1 to building 3 that go through exactly one other building.

Checkpoint 9

Use a graphing calculator to find the following.

- (a) A^3
(b) $A + A^2$

In the previous example, we could have also found the number of ways to walk between any two buildings by passing through exactly two other buildings by calculating A^3 . Additionally, $A + A^2$ gives the number of ways to walk between two buildings with at most one intermediate building.

The diagram can be given many other interpretations. For example, the lines could represent lines of mutual influence between people or nations, or they could represent communication lines, such as Ethernet cables.

6.6 Exercises

Solve the matrix equation $AX = B$ for X . (See Example 1.)

1. $A = \begin{bmatrix} 1 & -1 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 2 \\ 1 & 5 \end{bmatrix}, B = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$

3. $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}, B = \begin{bmatrix} 0 & 8 \\ 4 & 1 \end{bmatrix}$

$$5. A = \begin{bmatrix} 2 & 1 & 0 \\ -4 & -1 & 3 \\ 3 & 1 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$$

$$6. A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & 1 & 2 \\ 9 & 0 & 5 \end{bmatrix}, B = \begin{bmatrix} -3 \\ 6 \\ 12 \end{bmatrix}$$

Use the inverse of the coefficient matrix to solve each system of equations. (See Example 1.)

$$7. \begin{cases} x + 2y + 3z = 10 \\ 2x + 3y + 2z = 6 \\ -x - 2y - 4z = -1 \end{cases}$$

$$8. \begin{cases} -x + y - 3z = 3 \\ 2x + 4y - 4z = 6 \\ -x + y + 4z = -1 \end{cases}$$

$$9. \begin{cases} x + 4y + 3z = -12 \\ x - 3y - 2z = 0 \\ 2x + 5y + 4z = 7 \end{cases}$$

$$10. \begin{cases} 5x + 2z = 3 \\ 2x + 2y + z = 4 \\ -3x + y - z = 5 \end{cases}$$

$$11. \begin{cases} x + 2y + 3z = 4 \\ x + 4y + 2z = 8 \\ y - z = -4 \end{cases}$$

$$12. \begin{cases} 2x + 2y - 4z = 12 \\ 2x + 6y = 16 \\ -3x - 3y + 5z = -20 \end{cases}$$

$$13. \begin{cases} x + y + 2w = 3 \\ 2x - y + z - w = 3 \\ 3x + 3y + 2z - 2w = 5 \\ x + 2y + z = 3 \end{cases}$$

$$14. \begin{cases} x - 2z = 4 \\ -2x + y + 2z + 2w = -8 \\ 3x - y - 2z - 3w = 12 \\ y + 4z + w = -4 \end{cases}$$

Write a system of equations, and use the inverse of the coefficient matrix to solve each system of equations. (See Example 1.)

15. **E-Commerce** For online store sales, the payment processing service Square charges 2.9% of the total sales plus \$.30 per transaction. For your online business, the cost of using Square last month was \$291. A competitor to Square charges 3% of total sales plus \$.25 per transaction, and your bill for the same month with that company would have been \$295. Find the total sales amount and the number of transactions. (Data from: www.squareup.com.)

16. **Truck Rental** Budget Truck Rental charged \$19.99 per day plus \$.59 per mile to rent a 12-foot moving truck in Austin, Texas, in 2016. The total bill for your move was \$86. The rates for a local moving company were \$10 per day and \$.95 per mile, and the bill for same trip (same number of miles in the same number of days) would have been \$94.10. How many miles and how many days was this trip? (Data from: www.budgettruck.com.)

17. **Furniture Manufacturing** A Colorado-based company makes artisanal dining room furniture. A buffet requires 15 hours for cutting, 20 hours for assembly, and 5 hours for finishing. A chair requires 5 hours for cutting, 8 hours for assembly, and 5 hours for finishing. A table requires 10 hours for cutting, 6 hours for assembly, and 6 hours for finishing. The cutting department has 4900 hours of labor available each week, the assembly department has 6600 hours available, and the finishing department has 3900 hours available. How many pieces of each type of furniture should be produced each week if the factory is to run at full capacity?

18. **Nutrition** A hospital dietician is planning a special diet for a certain patient. The total amount per meal of food groups A, B,

and C must equal 400 grams. The diet should include one-third as much of group A as of group B, and the sum of the amounts of group A and group C should equal twice the amount of group B. How many grams of each food group should be included?

19. **Casual Clothing** A national chain of casual clothing stores recently sent shipments of jeans, jackets, sweaters, and shirts to its stores in various cities. The number of items shipped to each city and their total wholesale cost are shown in the table. Find the wholesale price of one pair of jeans, one jacket, one sweater, and one shirt.

City	Jeans	Jackets	Sweaters	Shirts	Total Cost
Cleveland	3000	3000	2200	4200	\$507,650
St. Louis	2700	2500	2100	4300	459,075
Seattle	5000	2000	1400	7500	541,225
Phoenix	7000	1800	600	8000	571,500

20. **Materials Mixing** A company produces three combinations of mixed vegetables, which sell in 1-kilogram packages. Italian style combines .3 kilogram of zucchini, .3 of broccoli, and .4 of carrots. French style combines .6 kilogram of broccoli and .4 of carrots. California style combines .2 kilogram of zucchini, .5 of broccoli, and .3 of carrots. The company has a stock of 16,200 kilograms of zucchini, 41,400 kilograms of broccoli, and 29,400 kilograms of carrots. How many packages of each style should the company prepare in order to use up its supplies?

21. **Bacteria Colonies** Three species of bacteria are fed three foods: I, II, and III. A bacterium of the first species consumes 1.3 units each of foods I and II and 2.3 units of food III each day. A bacterium of the second species consumes 1.1 units of food I, 2.4 units of food II, and 3.7 units of food III each day. A bacterium of the third species consumes 8.1 units of I, 2.9 units of II, and 5.1 units of III each day. If 16,000 units of I, 2500 units of II, and 44,000 units of III are supplied each day, how many of each species can be maintained in this environment?

22. **Assisted Living** A 100-bed assisted living facility provides two levels of long-term care: regular and maximum. Patients on each level have a choice of a private room or a less expensive semiprivate room. The tables (on this page and the following page) show the number of patients in each category at various times this year. The total daily costs for all patients were \$24,040 in January, \$23,926 in April, \$23,760 in July, and \$24,042 in October. Find the daily cost of each of the following: a private room (regular care), a private room (maximum care), a semiprivate room (regular care), and a semiprivate room (maximum care).

REGULAR-CARE PATIENTS		
Month	Semiprivate	Private
January	22	8
April	26	8
July	24	14
October	20	10

Month	MAXIMUM-CARE PATIENTS	
	Semiprivate	Private
January	60	10
April	54	12
July	56	6
October	62	8

Find the production matrix for the given input-output and demand matrices. (See Examples 3-6.)

$$21. \quad A = \begin{bmatrix} 1 & .03 \\ .07 & .6 \end{bmatrix}, D = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$24. \quad A = \begin{bmatrix} .7 & .5 \\ .2 & .4 \end{bmatrix}, D = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Exercises 25 and 26 refer to Example 6.

25. **Input-Output Analysis** If the demand is changed to 690 metric tons of wheat and 920 metric tons of oil, how many units of each commodity should be produced?

26. **Input-Output Analysis** Change the input-output matrix so that the production of 1 metric ton of wheat requires $\frac{1}{2}$ metric ton of oil (and no wheat) and the production of 1 metric ton of oil requires $\frac{1}{4}$ metric ton of wheat (and no oil). To satisfy the same demand matrix, how many units of each commodity should be produced?

Work these problems. (See Examples 3-6.)

27. **Input-Output Analysis** A primitive economy depends on two basic goods: yams and pork. The production of 1 bushel of yams requires $\frac{1}{4}$ bushel of yams and $\frac{1}{2}$ of a pig. To produce 1 pig requires $\frac{1}{4}$ bushel of yams. Find the amount of each commodity that should be produced to get

- 1 bushel of yams and 1 pig;
- 100 bushels of yams and 70 pigs.

28. **Input-Output Analysis** A two-segment economy consists of manufacturing and agriculture. To produce 1 unit of manufacturing output requires .40 unit of its own output and .20 unit of agricultural output. To produce 1 unit of agricultural output requires .30 unit of its own output and .40 unit of manufacturing output. If there is a demand of 240 units of manufacturing and 90 units of agriculture, what should be the output of each segment?

29. **Input-Output Analysis** A simplified economy has only two industries: the electric company and the gas company. Each dollar's worth of the electric company's output requires \$.40 of its own output and \$.50 of the gas company's output. Each dollar's worth of the gas company's output requires \$.25 of its own output and \$.60 of the electric company's output. What should the production of electricity and gas be (in dollars) if there is a \$12 million demand for gas and a \$15 million demand for electricity?

30. **Input-Output Analysis** A simplified economy is based on agriculture, manufacturing, and transportation. Each unit of agricultural output requires .4 unit of its own output, .3 unit of manufacturing output, and .2 unit of transportation output.

One unit of manufacturing output requires .4 unit of its own output, .2 unit of agricultural output, and .3 unit of transportation output. One unit of transportation output requires .4 unit of its own output, .1 unit of agricultural output, and .2 unit of manufacturing output. There is demand for 35 units of agricultural, 90 units of manufacturing, and 20 units of transportation output. How many units should each segment of the economy produce?

31. **Input-Output Analysis** In his work *Input-Output Economics*, Leontief provides an example of a simplified economy with just three sectors: agriculture, manufacturing, and households (that is, the sector of the economy that produces labor).^{*} It has the following input-output matrix:

	Agriculture	Manufacturing	Households
Agriculture	.25	.40	.133
Manufacturing	.14	.12	.100
Households	.80	3.60	.133

- How many units from each sector does the manufacturing sector require to produce 1 unit?
- What production levels are needed to meet a demand of 35 units of agriculture, 38 units of manufacturing, and 40 units of households?
- How many units of agriculture are used up in the economy's production process?

32. **Input-Output Analysis** A much simplified version of Leontief's 42-sector analysis of the American economy has the following input-output matrix:[†]

	Agriculture	Manufacturing	Households
Agriculture	.245	.102	.051
Manufacturing	.099	.291	.279
Households	.433	.372	.011

- What information about the needs of agricultural production is given by column 1 of the matrix?
- Suppose the demand matrix (in billions of dollars) is

$$D = \begin{bmatrix} 2.88 \\ 31.45 \\ 30.91 \end{bmatrix}.$$

Find the amount of each commodity that should be produced.

33. **Input-Output Analysis** An analysis of the Israeli economy is simplified here by grouping the economy into three sectors, with the following input-output matrix:[‡]

	Agriculture	Manufacturing	Energy
Agriculture	.293	0	0
Manufacturing	.014	.207	.017
Energy	.044	.010	.216

- How many units from each sector does the energy sector require to produce one unit?

^{*} Wassily Leontief, *Input-Output Economics*, 2d ed. (Oxford University Press, 1986), pp. 19-27.

[†] Ibid., pp. 6-9.

[‡] Ibid., pp. 174-175.

- (b) If the economy's production (in thousands of Israeli pounds) is 175,000 of agriculture, 22,000 of manufacturing, and 12,000 of energy, how much is available from each sector to satisfy the demand from consumers and others outside the production process?

(c) The actual demand matrix is

$$D = \begin{bmatrix} 138,213 \\ 17,597 \\ 1786 \end{bmatrix}$$

How much must each sector produce to meet this demand?

34. **Input-Output Analysis** The Chinese economy can be simplified to three sectors: agriculture, industry and construction, and transportation and commerce.* The input-output matrix is

	Agri.	Industry/Constr.	Trans./Commerce
Agri.	.158	.156	.009
Industry/Constr.	.136	.432	.071
Trans./Commerce	.013	.041	.011

The demand [in 100,000 renminbi (RMB), the unit of money in China] is

$$D = \begin{bmatrix} 106,674 \\ 144,739 \\ 26,725 \end{bmatrix}$$

- (a) Find the amount each sector should produce.
 (b) Interpret the economic value of an increase in demand of 1 RMB in agriculture exports.

35. **Input-Output Analysis** The economy of the state of Washington has been simplified to four sectors: natural resource, manufacturing, trade and services, and personal consumption. The input-output matrix is as follows:†

	Natural Resources	Manufacturing	Trade and Services	Personal Consumption
Natural Resources	.1045	.0428	.0029	.0031
Manufacturing	.0826	.1087	.0584	.0321
Trade and Services	.0867	.1019	.2032	.3555
Personal Consumption	.6253	.3448	.6106	.0798

Suppose the demand (in millions of dollars) is

$$D = \begin{bmatrix} 450 \\ 300 \\ 125 \\ 100 \end{bmatrix}$$

Find the amount each sector should produce.

*Input-Output Tables of China: 1981, China Statistical Information and Consultancy Service Centre, 1987, pp. 17-19.

†Robert Chase, Philip Bourque, and Richard Conway, Jr., *The 1987 Washington State Input-Output Study*, Report to the Graduate School of Business Administration, University of Washington, September 1993.

36. **Input-Output Analysis** The economy of the state of Nebraska has been condensed to six sectors: livestock, crops, food products, mining and manufacturing, households, and other. The input-output matrix is as follows:‡

	Livestock	Crops	Food Products	Mining and Manufacturing	Households	Other
Livestock	.178	.018	.411	0	.005	0
Crops	.143	.018	.088	0	.001	0
Food Products	.089	0	.035	0	.060	.003
Mining and Manufacturing	.001	.010	.012	.063	.007	.014
Households	.141	.252	.088	.089	.402	.124
Other	.188	.156	.103	.255	.008	.474

- (a) Find the matrix $(I - A)^{-1}$ and interpret the value in row two, column one, of this matrix.

(b) Suppose the demand (in millions of dollars) is

$$D = \begin{bmatrix} 1980 \\ 650 \\ 1750 \\ 1000 \\ 2500 \\ 3750 \end{bmatrix}$$

Find the dollar amount each sector should produce.

Work these coding exercises. (See Example 7 and 8.)

37. **Encoding** Encode the message.

Anne is home.

Break the message into groups of two letters and use the matrix

$$M = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

38. **Encoding** Use the matrix of Exercise 37 to encode the message

Head for the hills!

39. **Decoding** Decode the following message, which was encoded using the matrix M of Exercise 37:

$$\begin{bmatrix} 57 \\ 129 \end{bmatrix}, \begin{bmatrix} 37 \\ 79 \end{bmatrix}$$

40. **Decoding** Decode the following message, which was encoded by using the matrix M of Exercise 37:

$$\begin{bmatrix} 90 \\ 207 \end{bmatrix}, \begin{bmatrix} 39 \\ 87 \end{bmatrix}, \begin{bmatrix} 26 \\ 57 \end{bmatrix}, \begin{bmatrix} 66 \\ 145 \end{bmatrix}, \begin{bmatrix} 61 \\ 142 \end{bmatrix}, \begin{bmatrix} 89 \\ 205 \end{bmatrix}$$

Work these routing problems. (See Example 9.)

41. **Highway Networks** The matrix

$$\begin{matrix} & \begin{matrix} \text{City 1} & \text{City 2} & \text{City 3} & \text{City 4} \end{matrix} \\ \begin{matrix} \text{City 1} \\ \text{City 2} \\ \text{City 3} \\ \text{City 4} \end{matrix} & \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{bmatrix} \end{matrix} = I$$

‡F. Charles Lamphear and Theodore Rooster, "1976 Nebraska Input-Output Tables," *Nebraska Economic and Business Report No. 10*, Bureau of Business Research, University of Nebraska, Lincoln.

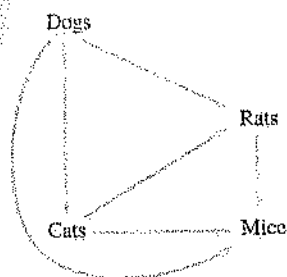
the number of roads between various cities. Find A^2 , then use it to answer the following questions: How many ways are there to travel from

- city 1 to city 3 by passing through exactly one city?
- city 2 to city 4 by passing through exactly one city?
- city 1 to city 3 by passing through at most one city?
- city 2 to city 4 by passing through at most one city?

42. **Highway Networks** For matrix A in the previous exercise, find A^2 , then use it to answer the following questions.

- How many ways are there to travel between cities 1 and 4 by passing through exactly two cities?
- How many ways are there to travel between cities 1 and 4 by passing through at most two cities?

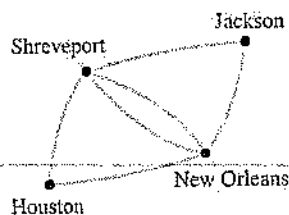
43. **Food Webs** The figure shows a food web. The arrows indicate the food sources of each population. For example, cats feed on rats and on mice.



(a) Write a matrix C in which each row and corresponding column represent a population in the food chain. Enter a 1 when the population in a given row feeds on the population in the given column.

(b) Calculate and interpret C^2 .

44. **Airline Routing** The figure shows four southern cities served by Supersouth Airlines.



- Write a matrix to represent the number of nonstop routes between cities.
- Find the number of one-stop flights between Houston and Jackson.
- Find the number of flights between Houston and Shreveport that require at most one stop.
- Find the number of one-stop flights between New Orleans and Houston.

Checkpoint Answers

1. $A = \begin{bmatrix} 2 & 6 \\ -1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, and $B = \begin{bmatrix} -14 \\ 3 \end{bmatrix}$.

2. $(6, -4, 10)$

3. $(3 - 1.5y, y)$ for all real numbers y

4. $\begin{matrix} \text{Elec.} & \text{Water} \\ \text{Elec.} & \begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & 0 \end{bmatrix} \end{matrix}$

5. (a) $\begin{bmatrix} 9000 \\ 12,000 \end{bmatrix}$ (b) $\begin{bmatrix} 6000 \\ 4500 \end{bmatrix}$ (c) $\begin{bmatrix} 3000 \\ 7500 \end{bmatrix}$

6. (a) $\begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ \frac{1}{2} & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & -\frac{3}{4} \\ -\frac{1}{2} & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 6 \\ 4 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 6800 \\ 4400 \end{bmatrix}$

7. $\begin{bmatrix} 23 \\ 8 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 14 \end{bmatrix}$

8. $\begin{bmatrix} 54 \\ 115 \end{bmatrix}$, $\begin{bmatrix} 24 \\ 25 \end{bmatrix}$

9. (a) $[A]^3 = \begin{bmatrix} 18 & 26 & 16 & 16 & 26 \\ 26 & 12 & 13 & 9 & 16 \\ 16 & 13 & 6 & 10 & 9 \\ 16 & 9 & 10 & 6 & 13 \\ 26 & 16 & 9 & 13 & 12 \end{bmatrix}$ (b) $[A] + [A]^2 = \begin{bmatrix} 10 & 5 & 4 & 4 & 6 \\ 5 & 6 & 3 & 4 & 5 \\ 4 & 3 & 3 & 2 & 4 \\ 4 & 4 & 2 & 3 & 3 \\ 5 & 5 & 4 & 3 & 6 \end{bmatrix}$

CHAPTER 6 Summary and Review

Key Terms and Symbols

6.1 linear equation
system of linear equations
solution of a system
substitution method
elimination method
independent system

dependent system
inconsistent system
6.2 equivalent systems
elementary operations
elimination method
row

column
matrix (matrices)
element (entry)
augmented matrix
row operations
row echelon form

Gauss-Jordan method
reduced row echelon form
independent system
inconsistent system
dependent system
parameter